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ARTUR PAWEŁ POLIŃSKI

ON SELECTED ISSUES OF NON-INVASIVE BLOOD PRESSURE ESTIMATION

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Symbols

a	–	auxiliary parameter
a_1	–	a constant
a_2	–	a constant
a_3	–	a constant
a_4	–	a constant
a_{i_1}	–	a constant
a_{i_1,i_2}	–	a constant
$adiff$	–	fiducial point of the signal defined by the maximum of its derivative using approximation
A	–	vessel cross section
A_0	–	vessel cross-section for $p = p_0$
b	–	auxiliary parameter
b_1	–	a constant
b_2	–	a constant
b_3	–	a constant
b_{i_1}	–	a constant
b_{i_1,i_2}	–	a constant
c	–	wave speed
c_0	–	reference wave speed
c_{i_1}	–	a constant
c_{i_2}	–	wave speed
c_{i_1,i_2}	–	a constant
c_{i_1,i_2,i_3}	–	a constant
C	–	capacitance
CUS	–	artificial neural network with custom activation function
C_0	–	capacitance
C_1	–	a constant
C_2	–	a constant
C_3	–	a constant
C_4	–	a constant
C_c	–	a constant
C_{init}	–	initial value of auxiliary constant C_c
C_o	–	capacitance
C_T	–	capacitance
d	–	a constant
da	–	disturbance of the a

db	–	disturbance of the b
$diff$	–	fiducial point of the signal defined by the maximum of its derivative
ds_{min}	–	disturbance of the minimum of the signal s_{min}
dt	–	time discretization of the signal
d_1	–	a constant
d_2	–	a constant
D	–	vessel length
D_1	–	vessel length
D_2	–	vessel length
D_i	–	axillary variable related to vessel length
D_{i_2}	–	vessel length
D_{i_a}	–	axillary variable related to some location along the vessel
D_{i_b}	–	axillary variable related to some location along the vessel
D_{n_s}	–	vessel length
e_1	–	the error measure based on the maximum norm
e_2	–	the error measure based on the 1 norm
e_3	–	the error measure based on the 2 norm
E	–	Young modulus
EX	–	expected value of X
E_k	–	electrode location
f	–	frequency
f_c	–	frequency
f_s	–	sampling frequency
F	–	inverse function of g
$FF1$	–	one-layer feedforward neural network
$FF2$	–	three-layer feedforward neural network
g	–	a function
g^{-1}	–	inverse of function g
g_1	–	piecewise linear function
g_2	–	a function
G	–	conductance
h	–	transfer function
h_w	–	thickness of vessel wall
h_0	–	thickness of vessel wall for p_0
H	–	Fourier transform of h
i	–	current
i_1	–	auxiliary variable
i_2	–	auxiliary variable
i_3	–	auxiliary variable
j	–	imaginary unit
k	–	auxiliary variable
k_1	–	a constant
k_2	–	a constant
k_3	–	a constant
K_1	–	a constant
K_2	–	a constant

K_3	–	a constant
K_4	–	a constant
l	–	distance between electrodes
l_b	–	length of resistor representing blood
l_m	–	length of resistor representing muscle
l_R	–	length of resistor
L	–	inductance
L_s	–	distance between measurement sensors
m	–	a constant
m_1	–	a constant
m_2	–	a constant
min	–	fiducial point of the signal defined by its minimum
max	–	fiducial point of the signal defined by its maximum
M	–	auxiliary variable
M_1	–	auxiliary variable
M_{1init}	–	initial value of auxiliary constant M_1
M_2	–	auxiliary variable
M_{2init}	–	initial value of auxiliary constant M_2
n	–	discrete time moment
n_s	–	number of vessel segments
n_v	–	normal vector
N	–	length of the model
N_d	–	length of the data
N_i	–	number of inputs of artificial neural network
N_p	–	number of model parameters
N_s	–	number of samples
p	–	pressure
p_0	–	reference pressure
p_{end}	–	pressure at the end of the vessel
p_T	–	pressure at the venous end of the arterial tree
P	–	probability
PTTcp	–	pulse transit time defined for the same fiducial point
PTTbp	–	pulse transit time defined for the same value of blood pressure
q	–	blood flow
q_{end}	–	blood flow at the end of the vessel
qm_1	–	quality measure based on correlation coefficient
qm_2	–	quality measure based on the 2 norm
qm_3	–	quality measure based on the 2 norm and penalty of the number of model parameters and data length
$resp$	–	value of respiration signal at the moment of R-wave of the electrocardiography signal
r_0	–	vessel radius for $p = p_0$
r_{in}	–	initial radius of the vessel
r_{out}	–	ending radius of the vessel
R	–	resistance
R_1	–	resistance

R_2 – resistance
R_o – resistance
RR – time distance between two successive R-waves of the electrocardiography signal
$RNN1$ – one-layer recurrent neural network
$RNN3$ – three-layer recurrent neural network
s – signal
s_1 – signal
s_2 – signal
s_{min} – minimum of the signal
S – Fourier transform of s
S_f – sensitivity function
S_1 – Fourier transform of s_1
S_2 – Fourier transform of s_2
S_a – area of cross-section
S_b – area of blood vessel cross-section
S_m – area of muscle cross-section
t – time
t_0 – time shift
t_1 – a time moment
t_2 – a time moment
t_{min} – time of minimum signal occurrence
t_{max} – time of maximum signal occurrence
tg – fiducial point of the signal defined by tangent intersection
T – maximum considered moment of time
TDB – the duration from the maximum derivative point to the maximum of the dicrotic notch in the photoplethysmography signal
u – voltage
v – phase velocity
V – volume
V_b – blood volume
V_m – muscle volume
w_0 – a weight
w_{i_1} – a weight
w_{i_2} – a weight
x – space variable
x_1 – selected space variable
x_2 – selected space variable
x_v – auxiliary parameter
x_{vi_1} – auxiliary parameter
X – random variable
y – space variable
y_v – auxiliary variable
z – space variable
Z – impedance
Z_c – characteristic impedance

Z_o	–	output impedance
α	–	auxiliary parameter
α_0	–	auxiliary parameter
β	–	phase constant
β_{i-1}	–	some constant
γ	=	$(1+\Gamma)/(1-\Gamma)$
Γ	–	reflection coefficient
Δx_v	–	change in x_v
ϵ	–	permittivity
λ	–	parameter describing ex-Gaussian model
λ_{init}	–	initial value of λ
μ_0	–	expected value
μ_1	–	expected value
μ_{1init}	–	initial value of μ_1
μ_2	–	expected value
μ_{2init}	–	initial value of μ_2
μ_{bv}	–	blood viscosity
ρ	–	resistivity
ρ_b	–	blood resistivity
ρ_{bd}	–	blood density
ρ_l	–	resistivity in longitudinal direction
ρ_{ml}	–	muscle resistivity in longitudinal direction
ρ_{mt}	–	muscle resistivity in transverse direction
ρ_t	–	resistivity in transverse direction
σ	–	conductivity
σ_0	–	standard deviation
σ_1	–	standard deviation
σ_{1init}	–	initial value of σ_1
σ_2	–	standard deviation
σ_{2init}	–	initial value of σ_2
σ_l	–	conductivity in longitudinal direction
σ_t	–	conductivity in transverse direction
ϕ	–	potential distribution
ψ	–	potential distribution
ω	=	$2\pi f$
ω_c	=	$2\pi f_c$
Ω	–	space domain
$\partial\Omega$	–	boundary of space domain

Acronyms

AIC	–	Akaike Information Criterion
ANN	–	Artificial Neural Network
BIC	–	Bayesian Information Criterion
BP	–	Blood Pressure
DBP	–	Diastolic Blood Pressure
ECG	–	Electrocardiography
FEM	–	Finite Element Method
FIR	–	Finite Impulse Response
GRU	–	Gated Recurrent Unit
HR	–	Heart Rate
LP	–	low-pass filter
MAE	–	Mean Absolute Error
MAP	–	Mean Arterial Pressure
MDL	–	Minimum Description Length
MSE	–	Mean Squared Error
PAT	–	Pulse Arrival Time
PAT1	–	PAT defined by the minimum of the signal
PAT2	–	PAT defined by tangent intersection
PAT3	–	PAT defined by the maximum of the first derivative of the signal
PAT4	–	PAT defined by the maximum of the signal
PEP	–	Pre-Ejection Period
PP	–	Pulse Pressure
PPG	–	Photoplethysmography
PTT	–	Pulse Transit Time
PWV	–	Pulse Wave Velocity
ReLU	–	Rectified Linear Unit
RGB	–	Red Green Blue
RMS	–	Root Mean Square
RMSE	–	Root Mean Square Error
RNN	–	Recurrent Neural Networks
SBP	–	Systolic Blood Pressure
std	–	standard deviation

Glossary of Key Terms

cardiac output (CO)
a volume of blood pumped by the heart into the systemic circulation per minute

characteristic impedance(Z_c)
a ratio of voltage to current of a forward-propagating wave in electrical engineering, and in arterial modelling the corresponding impedance of an arterial segment, determined by its stiffness and geometric properties

characteristic (fiducial) point
a specific location on a physiological waveform, defined by a reproducible signal feature such as a minimum, maximum, ect., used in blood pressure estimation from signals like PPG, IPG, or related modalities

compliance
a measure of the vessel's ability to change its volume when intravascular pressure varies.

impedance plethysmography (IPG)
a technique that measures changes in electrical impedance resulting from variations in tissue conductivity, typically caused by fluctuations in blood volume

phase velocity (v)
an apparent velocity at which a constant-phase point of a sinusoidal wave propagates along a transmission line

photoplethysmography (PPG)
a technique that measures changes in the intensity of transmitted or reflected light caused by pulsatile variations in blood volume within tissue

pre-ejection period (PEP)
a time interval between the onset of ventricular depolarization and the opening of the aortic valve, typically identified from impedance cardiography or other hemodynamic signals

pulse arrival time (PAT)
a time interval between the R wave of the electrocardiogram and a characteristic point of a peripheral measure by PPG, ICG or similar technique

pulse pressure (PP)
a difference between systolic and diastolic pressure, reflecting arterial stiffness and stroke volume

pulse transit time (PTT)
a time interval between two characteristic points of cardiovascular waveforms recorded at different sites along the arterial tree, typically using PPG, ICG, or similar techniques

pulse wave velocity (PWV)
a speed at which the arterial pressure wave travels between two sites along the arterial system

recurrent neural networks (RNN)
a class of neural architectures designed to process data sequences of arbitrary length, such as time series or text, by maintaining internal states that capture temporal dependencies

reflection coefficient (Γ)
a ratio of reflected to incident wave amplitude at an arterial impedance discontinuity

stiffness
a measure of the reduced ability of an artery to expand and recoil when blood pressure changes

Windkessel model
a lumped-parameter representation of the cardiovascular system that describes hemodynamics using RLC elements to capture vascular resistance, compliance, and inertance

Introduction

An individual's blood pressure is a critical parameter for assessing overall health. It allows for the early detection of potential issues, such as hypertension or hypotension, which can lead to severe conditions like heart disease, stroke, or kidney failure. Hence, it is desirable to develop a method that will allow blood pressure to be measured continuously and non-invasively, which will not be burdensome for the tested person. There are different approaches to achieve this goal. Thus, in this work, we will focus on selected techniques and problems related to non-invasive blood pressure estimation.

The monograph presents a coherent and comprehensive description of the non-invasive blood pressure estimation. It extends and deepens the author's previous research results for this research problem.

The first chapter introduces different methods applicable to non-invasive blood pressure estimation.

Chapter 2 contains novel considerations on blood flow modelling. It is shown that mathematical modelling is a useful tool for analysing complex physiological phenomena, enabling a deeper understanding of the problem and the formulation and verification of potential practical solutions. In particular, the influence of model parameters on the results is analysed, and the limitations of modelling complex biological systems are identified.

Chapter 3 examines the impact of sampling frequency and low-pass filtering on the accuracy of detecting characteristic points in signals. In contrast to previous studies, the analysis is performed on blood pressure rather than photoplethysmographic signals, thereby eliminating the influence of the unknown transfer function between the these signals. This enables a more precise identification of limitations arising from the sampling and filtering processes themselves. As an alternative to classical filtering, a signal-modelling-based approach is proposed, and its limitations are analysed. Compared to earlier studies, the analysis is extended to include simulations involving a broader, more diverse class of signals, and additional approximation functions are considered.

Chapter 4 is devoted to the analysis of the photoplethysmographic signal in both the time and frequency domains. Relationships describing the connection between blood pressure and pulse transit time are systematised, and the physiological foundations of these relationships, resulting from blood volume–induced pressure changes in the vascular system, are discussed. The chapter extends previous modelling analyses and presents an original examination of the limitations of frequency-domain-based approaches.

Chapter 5 presents novel analyses of impedance-based models that enable identification of sources of impedance change signals, depending on electrode configuration.

Chapter 6 presents an analysis of physiologically derived models for blood pressure estimation that, in addition to pulse arrival time, incorporate signals such as ECG and respiration. The study provides a fully elaborated treatment, including explicit derivations and enhanced analytical detail, building upon the original conceptual framework.

Chapter 7 addresses the use of neural networks' approximation capabilities. Compared to previous studies, the scope of analysis is extended to a larger number of model architectures. A comparison between neural network models and physiologically derived models is also conducted, assuming identical input data and a comparable number of parameters. This allows for the assessment of the extent to which limitations in accuracy result from the inadequacy of physiological models rather than from the limited information content of the available data.

The final chapter presents an analysis of two research problems. The first, concerning the locality of pulse wave velocity, is documented in the literature but has not been studied in detail to date. The second complements previous investigations and concerns the influence of pulse waveform deformation along the vascular tree.